# COMPLEX NUMBERS AND DE MOIVRE'S THEOREM

## OBJECTIVE PROBLEMS

**1.** $\left(\frac{1+i}{1-i}\right)^2+\left(\frac{1-i}{1+i}\right)^2$ **is equal to**

(a) $2i$ (b) $-2i$

(c) $-2$ (d) $2$

**2.** $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)=$

(a) $\frac{1}{2}+\frac{9}{2}i$ (b) $\frac{1}{2}-\frac{9}{2}i$

(c) $\frac{1}{4}-\frac{9}{4}i$ (d) $\frac{1}{4}+\frac{9}{4}i$

**3. The imaginary part of** $\frac{(1+i)^2}{(2-i)}$ **is**

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) None of these

**4. If** $\left(\frac{1+i}{1-i}\right)^m=1$, **then the least integral value of** $m$ **is**

(a) 2 (b) 4

(c) 8 (d) None of these

**5. The value of** $i^{1+3+5+...+(2n+1)}$ **is**

(a) $i$ if $n$ is even, $-i$ if $n$ is odd

(b) 1 if $n$ is even, – 1 if $n$ is odd

(c) 1 if $n$ is odd, – 1 if $n$ is even

(d) $i$ if $n$ is even, – 1 if $n$ is odd

**6. The real values of** $x$ **and** $y$ **for which the equation is** $(x+iy)(2-3i)=4+i$ **is satisfied, are**

(a) $x=\frac{5}{13}, y=\frac{8}{13}$ (b) $x=\frac{8}{13}, y=\frac{5}{13}$

(c) $x=\frac{5}{13}, y=\frac{14}{13}$ (d) None of these

**7.** $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ **will be real, if** $\theta$ **=**

(a) $2n\pi$ (b) $n\pi+\frac{\pi}{2}$

(c) $n\pi$ (d) None of these

**8. If** $a=\cos\theta+i\sin\theta,$ **then** $\frac{1+a}{1-a}=$

(a) $\cot\theta$ (b) $\cot\frac{\theta}{2}$

(c) $i\cot\frac{\theta}{2}$ (d) $i\tan\frac{\theta}{2}$

**9. The multiplication inverse of a number is the number itself, then its initial value is**

(a) $i$ (b) – 1

(c) 2 (d) – $i$

**10. If** $z_1=1-i$ **and** $z_2=-2+4i$**, then** $\text{Im}\left(\frac{z_1z_2}{z_1}\right)=$

(a) 1 (b) 2

(c) 3 (d) 4

**11. If** $(x+iy)^{1/3}=a+ib,$ **then** $\frac{x}{a}+\frac{y}{b}$ **is equal to**

(a) $4(a^2+b^2)$ (b) $4(a^2-b^2)$

(c) $4(b^2-a^2)$ (d) None of these

**12. The values of** $x$ **and** $y$ **satisfying the equation** $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ **are**

(a) $x=-1, y=3$ (b) $x=3, y=-1$

(c) $x=0, y=1$ (d) $x=1, y=0$

**13.** $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ **will be purely imaginary, if** $\theta=$

(a) $2n\pi\pm\frac{\pi}{3}$ (b) $n\pi+\frac{\pi}{3}$

(c) $n\pi\pm\frac{\pi}{3}$ (d) None of these

**14. If** $(1-i)^n=2^n$**, then** $n=$

(a) 1 (b) 0

(c) $-1$ (d) None of these

15. **The smallest positive integer** $n$ **for which** $(1+i)^{2n} = (1-i)^{2n}$ **is**

(a) 1 (b) 2

(c) 3 (d) 4

16. **The real part of** $(1-\cos\theta + 2i\sin\theta)^{-1}$ **is**

(a) $\frac{1}{3+5\cos\theta}$ (b) $\frac{1}{5-3\cos\theta}$ (c) $\frac{1}{3-5\cos\theta}$ (d) $\frac{1}{5+3\cos\theta}$

17. **If** $z = 1+i$, **then the multiplicative inverse of** $z^2$ **is (where** $i = \sqrt{-1}$**)**

(a) $2i$ (b) $1-i$

(c) $-i/2$ (d) $i/2$

18. **If** $x+iy = \frac{3}{2+\cos\theta + i\sin\theta}$, **then** $x^2+y^2$ **is equal to**

(a) $3x-4$ (b) $4x-3$

(c) $4x+3$ (d) None of these

19. **If** $z(1+a) = b+ic$ **and** $a^2+b^2+c^2=1$, **then** $\frac{1+iz}{1-iz} =$

(a) $\frac{a+ib}{1+c}$ (b) $\frac{b-ic}{1+a}$

(c) $\frac{a+ic}{1+b}$ (d) None of these

20. **If** $(x+iy)(p+iq) = (x^2+y^2)i$, **then**

(a) $p=x, q=y$ (b) $p=x^2,\ q=y^2$

(c) $x=q, y=p$ (d) None of these

21. **If** $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x+iy$, **then (*x*, *y*) is**

(a) (3, 1) (b) (1, 3)

(c) (0, 3) (d) (0, 0)

22. **If** $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$, **then**

(a) $a=2, b=-1$ (b) $a=1, b=0$

(c) $a=0, b=1$ (d) $a=-1, b=2$

23. **If** $a^2+b^2=1$, **then** $\frac{1+b+ia}{1+b-ia} =$

(a) 1 (b) 2

(c) $b+ia$ (d) $a+ib$

**24. If $z=3-4i$, then $z^4-3z^3+3z^2+99z-95$ is equal to**

(a) 5 (b) 6

(c) – 5 (d) – 4

**25. The conjugate of $\frac{(2+i)^2}{3+i}$, in the form of *a* + *ib*, is**

(a) $\frac{13}{2}+i\left(\frac{15}{2}\right)$ (b) $\frac{13}{10}+i\left(\frac{-15}{2}\right)$ (c) $\frac{13}{10}+i\left(\frac{-9}{10}\right)$ (d) $\frac{13}{10}+i\left(\frac{9}{10}\right)$

**26. If $z_1$ and $z_2$ be complex numbers such that $z_1 \neq z_2$ and $|z_1|=|z_2|$. If $z_1$ has positive real part and $z_2$ has negative imaginary part, then $\frac{(z_1+z_2)}{(z_1-z_2)}$ may be**

(a) Purely imaginary (b) Real and positive

(c) Real and negative (d) None of these

**27. The maximum value of $|z|$ where *z* satisfies the condition $\left|z+\frac{2}{z}\right|=2$ is**

(a) $\sqrt{3}-1$ (b) $\sqrt{3}+1$

(c) $\sqrt{3}$ (d) $\sqrt{2}+\sqrt{3}$

**28. If $z$ is a complex number, then $(\overline{z^{-1}})(\bar{z})=$**

(a) 1 (b) –1

(c) 0 (d) None of these

**29. If** $z=3+5i$, then $z^3+\bar{z}+198=$

(a) $-3-5i$ (b) $-3+5i$

(c) $3+5i$ (d) $3-5i$

**30. The number of solutions of the equation $z^2+\bar{z}=0$ is**

(a) 1 (b) 2

(c) 3 (d) 4

**31. If $z_1$, $z_2$ are any two complex numbers, then $|z_1+\sqrt{z_1^2-z_2^2}|+|z_1-\sqrt{z_1^2-z_2^2}|$ is equal to**

(a) $|z_1|$ (b) $|z_2|$

(c) $|z_1+z_2|$ (d) $|z_1+z_2|+|z_1-z_2|$

**32.** **If** $z_1$ **and** $z_2$ **are two non-zero complex numbers such that** $|z_1+z_2|=|z_1|+|z_2|$, **then** $arg(z_1)-arg(z_2)$ **is equal to**

(a) $-\pi$ (b) $-\frac{\pi}{2}$

(c) $\frac{\pi}{2}$ (d) 0

**33.** **If** $z=1-\cos\alpha+i\sin\alpha$, **then** ***amp*** $z$ **=**

(a) $\frac{\alpha}{2}$ (b) $-\frac{\alpha}{2}$

(c) $\frac{\pi}{2}+\frac{\alpha}{2}$ (d) $\frac{\pi}{2}-\frac{\alpha}{2}$

**34.** **If** $z=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$ **then**

(a) $|z|=1,\ arg\,z=\frac{\pi}{4}$ (b) $|z|=1, arg\,z=\frac{\pi}{6}$

(c) $|z|=\frac{\sqrt{3}}{2}, arg\,z=\frac{5\pi}{24}$ (d) $|z|=\frac{\sqrt{3}}{2},\ arg z=\tan^{-1}\frac{1}{\sqrt{2}}$

**35.** **If** $|z_1|=|z_2|$ **and** $amp\,z_1+amp\,z_2=0$, **then**

(a) $z_1=z_2$ (b) $\bar{z}_1=z_2$

(c) $z_1+z_2=0$ (d) $\bar{z}_1=\bar{z}_2$

**36.** **If** $arg(z)=\theta$, **then** $arg(\bar{z})=$

(a) $\theta$ (b) $-\theta$

(c) $\pi-\theta$ (d) $\theta-\pi$

**37.** **If** $\bar{z}$ **be the conjugate of the complex number** $z$, **then which of the following relations is false**

(a) $|z|=|\bar{z}|$ (b) $z.\bar{z}=|\bar{z}|^2$

(c) $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$ (d) $arg\,z=arg\,\bar{z}$

**38.** **If** $|z_1|=|z_2|=.........=|z_n|=1$, **then the value of** $|z_1+z_2+z_3+..........+z_n|$ **=**

(a) 1 (b) $|z_1|+|z_2|+.......+|z_n|$

(c) $\left|\frac{1}{z_1}+\frac{1}{z_2}+.........+\frac{1}{z_n}\right|$ (d) None of these

**39. The conjugate of complex number $\frac{2-3i}{4-i}$, is**

(a) $\frac{3i}{4}$ (b) $\frac{11+10i}{17}$

(c) $\frac{11-10i}{17}$ (d) $\frac{2+3i}{4i}$

**40. If $z_1$ and $z_2$ are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right|$=1, then $\frac{z_1}{z_2}$ is a number which is**

(a) Positive real (b)Negative real

(c) Zero or purely imaginary (d) None of these

**41. If $z_1, z_2$ are two complex numbers such that $\left|\frac{z_1-z_2}{z_1+z_2}\right|=1$ and $iz_1=kz_2$, where $k\in R$, then the angle between $z_1-z_2$ and $z_1+z_2$ is**

(a) $\tan^{-1}\left(\frac{2k}{k^2+1}\right)$ (b) $\tan^{-1}\left(\frac{2k}{1-k^2}\right)$

(c) $-2\tan^{-1}k$ (d) $2\tan^{-1}k$

**42. If $|z|=1$ and $\omega=\frac{z-1}{z+1}$ (where $z\neq-1$), then $\text{Re}(\omega)$ is**

(a) $0$ (b) $-\frac{1}{|z+1|^2}$

(c) $\left|\frac{z}{z+1}\right|.\frac{1}{|z+1|^2}$ (d) $\frac{\sqrt{2}}{|z+1|^2}$

**43. If $\bar{z}$ be the conjugate of the complex number $z$, then which of the following relations is false**

(a) $|z|=|\bar{z}|$ (b) $z.\bar{z}=|\bar{z}|^2$

(c) $\overline{z_1+z_2}=\bar{z_1}+\bar{z_2}$ (d) $arg\, z=arg\,\bar{z}$

**44. Let $z_1$ be a complex number with $|z_1|=1$ and $z_2$ be any complex number, then $\left|\frac{z_1-z_2}{1-z_1\bar{z}_2}\right|=$**

(a) 0 (b) 1

(c) – 1 (d) 2

**45. For any two complex numbers** $z_1, z_2$ **we have** $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2$ **then**

(a) $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$  (b) $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$

(c) $\text{Re}(z_1 z_2) = 0$  (d) $\text{Im}(z_1 z_2) = 0$

**46. If** $|z_1+z_2| = |z_1-z_2|$**, then the difference in the amplitudes of** $z_1$ **and** $z_2$ **is**

(a) $\frac{\pi}{4}$  (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$  (d) 0

**47. If** $z$ **and** $\omega$ **are two non–zero complex numbers such that** $|z\omega| = 1$ **and** $arg(z) - arg(\omega) = \frac{\pi}{2}$, **then** $\bar{z}\omega$ **is equal to**

(a) 1  (b) – 1  (c) $i$  (d) – $i$

**48.** $arg\left(\frac{3+i}{2-i} + \frac{3-i}{2+i}\right)$ **is equal to**

(a) $\frac{\pi}{2}$  (b) $-\frac{\pi}{2}$

(c) 0  (d) $\frac{\pi}{4}$

**49. If** $z_1.z_2........z_n = z$, **then** $arg\, z_1 + arg\, z_2 + .... + arg\, z_n$ **and** $arg\, z$ **differ by a**

(a) Multiple of $\pi$  (b) Multiple of $\frac{\pi}{2}$

(c) Greater than $\pi$  (d) Less than 2 $\pi$

**50. Which of the following are correct for any two complex numbers** $z_1$ **and** $z_2$

(a) $|z_1 z_2| = |z_1||z_2|$  (b) $arg\,(z_1 z_2) = (arg\, z_1)(arg\, z_2)$

(c) $|z_1+z_2| = |z_1| + |z_2|$  (d) $|z_1 - z_2| \geq |z_1| - |z_2|$

**51. If** $z_1 = 1+2i$ **and** $z_2 = 3+5i$**, and then** $\text{Re}\left(\frac{\bar{z}_2 z_1}{z_2}\right)$ **is equal to**

(a) $\frac{-31}{17}$  (b) $\frac{17}{22}$

(c) $\frac{-17}{31}$  (d) $\frac{22}{17}$

**52. If** $(\sqrt{8}+i)^{50} = 3^{49}(a+ib)$ **then** $a^2+b^2$ **is**

(a) 3  (b) 8

(c) 9  (d) $\sqrt{8}$

**53. If** $|z_1| = |z_2|$ **and** $arg\left(\frac{z_1}{z_2}\right) = \pi$**, then** $z_1 + z_2$ **is equal to**

(a) 0 (b) Purely imaginary

(c) Purely real (d) None of these

**54. The real part of** $(1-i)^{-i}$ **is**

(a) $e^{-\pi/4}\cos\left(\frac{1}{2}\log 2\right)$ (b) $-e^{-\pi/4}\sin\left(\frac{1}{2}\log 2\right)$

(c) $e^{\pi/4}\cos\left(\frac{1}{2}\log 2\right)$ (d) $e^{-\pi/4}\sin\left(\frac{1}{2}\log 2\right)$

**55. If** $\sqrt{a+ib} = x+iy$**, then possible value of** $\sqrt{a-ib}$ **is**

(a) $x^2+y^2$ (b) $\sqrt{x^2+y^2}$

(c) $x+iy$ (d) $x-iy$

**56.** $i\log\left(\frac{x-i}{x+i}\right)$ **is equal to**

(a) $\pi + 2\tan^{-1}x$ (b) $\pi - 2\tan^{-1}x$

(c) $-\pi + 2\tan^{-1}x$ (d) $-\pi - 2\tan^{-1}x$

**57.** $\frac{1+7i}{(2-i)^2} =$

(a) $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ (b) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(c) $\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ (d) None of these

**58. The value of** $(-i)^{1/3}$ **is**

(a) $\frac{1+\sqrt{3}i}{2}$ (b) $\frac{1-\sqrt{3}i}{2}$

(c) $\frac{-\sqrt{3}-i}{2}$ (d) $-\frac{\sqrt{3}-i}{2}$

**59. If** $x+iy = \sqrt{\frac{a+ib}{c+id}}$**, then** $(x^2+y^2)^2 =$

(a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{a+b}{c+d}$

(c) $\frac{c^2+d^2}{a^2+b^2}$ (d) $\left(\frac{a^2+b^2}{c^2+d^2}\right)^2$

**60. If $(1+i\sqrt{3})^9 = a+ib,$ then $b$ is equal to**

(a) 1 (b) 256

(c) 0 (d) $9^3$

**61. The number of non-zero integral solutions of the equation $|1-i|^x = 2^x$ is**

(a) Infinite (b) 1

(c) 2 (d) None of these

**62. The imaginary part of $\tan^{-1}\left(\frac{5i}{3}\right)$ is**

(a) 0 (b) $\infty$

(c) $\log 2$ (d) $\log 4$

**63. If $y = \cos\theta + i\sin\theta$ ,then the value of $y+\frac{1}{y}$ is**

(a) $2\cos\theta$ (b) $2\sin\theta$

(c) $2\text{cosec}\,\theta$ (d) $2\tan\theta$

**64. $\frac{1-i}{1+i}$ is equal to**

(a) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (b) $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$

(c) $\sin\frac{\pi}{2}+i\cos\frac{\pi}{2}$ (d) None of these

**65. If $|z^2-1| = |z|^2+1$, then $z$ lies on**

(a) An ellipse (b) The imaginary axis

(c) A circle (d) The real axis

**66. The equation $z\bar{z}+a\bar{z}+\bar{a}z+b=0, b\in R$ represents a circle if**

(a) $|a|^2 = b$ (b) $|a|^2 > b$

(c) $|a|^2 < b$ (d) None of these

**67. Let $a$ be a complex number such that $|a|<1$ and $z_1, z_2, ......$ be vertices of a polygon such that $z_k = 1+a+a^2+.....+a^{k-1}$. Then the vertices of the polygon lie within a circle**

(a) $|z-a| = a$ (b) $\left|z-\frac{1}{1-a}\right| = |1-a|$

(c) $\left|z-\frac{1}{1-a}\right| = \frac{1}{|1-a|}$ (d) $|z-(1-a)| = |1-a|$

**68. Let the complex numbers $z_1, z_2$ and $z_3$ be the vertices of an equilateral triangle. Let $z_0$ be the circum centre of the triangle, then $z_1^2 + z_2^2 + z_3^2 =$**

(a) $z_0^2$ (b) $-z_0^2$

(c) $3z_0^2$ (d) $-3z_0^2$

**69. The complex numbers $z_1, z_2, z_3$ are the vertices of a triangle. Then the complex numbers $z$ which make the triangle into a parallelogram is**

(a) $z_1 + z_2 - z_3$ (b) $z_1 - z_2 + z_3$

(c) $z_2 + z_3 - z_1$ (d) All the above

**70. Let $z_1, z_2, z_3$ be three vertices of an equilateral triangle circumscribing the circle $|z| = \frac{1}{2}$. If $z_1 = \frac{1}{2} + \frac{\sqrt{3}\, i}{2}$ and $z_1, z_2, z_3$ are in anticlockwise sense then $z_2$ is**

(a) $1 + \sqrt{3}\, i$ (b) $1 - \sqrt{3}\, i$

(c) 1 (d) – 1

**71. If *z* is a complex number in the Argand plane, then the equation $|z-2| + |z+2| = 8$ represents**

(a) Parabola (b) Ellipse

(c) Hyperbola (d) Circle

**72. If $z_1, z_2, z_3, z_4$ are the affixes of four points in the Argand plane and $z$ is the affix of a point such that $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$, then $z_1, z_2, z_3, z_4$ are**

(a) Concyclic

(b) Vertices of a parallelogram

(c) Vertices of a rhombus

(d) In a straight line

**73. If $z = x + iy$, then area of the triangle whose vertices are points $z, iz$ and $z + iz$ is**

(a) $2|z|^2$ (b) $\frac{1}{2}|z|^2$ (c) $|z|^2$ (d) $\frac{3}{2}|z|^2$

**74. If $|z+1| = \sqrt{2}|z-1|$, then the locus described by the point $z$ in the Argand diagram is a**

(a) Straight line (b) Circle

**(c)** Parabola (d) None of these

**75. If the area of the triangle formed by the points $z, z+iz$ and *iz* on the complex plane is 18, then the value of $|z|$ is**

(a) 6 (b) 9 (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

**76. The region of Argand plane defined by $|z-1|+|z+1| \le 4$ is**

(a) Interior of an ellipse

(b) Exterior of a circle

(c) Interior and boundary of an ellipse

(d) None of these

**77. Let $z_1$ and $z_2$ be two complex numbers such that $\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$. Then**

(a) $z_1, z_2$ are collinear

(b) $z_1, z_2$ and the origin form a right angled triangle

(c) $z_1, z_2$ and the origin form an equilateral triangle

(d) None of these

**78. The locus represented by $|z-1|=|z+i|$ is**

(a) A circle of radius 1

(b) An ellipse with foci at $(1,0)$ and (0, – 1)

(c) A straight line through the origin

(d) A circle on the line joining $(1,0),(0,1)$ as diameter

**79. If $z = x+iy$ and $|z-2+i|=|z-3-i|$, then locus of *z* is**

(a) $2x+4y-5=0$ (b) $2x-4y-5=0$

(c) $x+2y=0$ (d) $x-2y+5=0$

**80.** $\left(\frac{1+\cos\varphi+i\sin\varphi}{1+\cos\varphi-i\sin\varphi}\right)^n =$

(a) $\cos n\phi - i\sin n\phi$ (b) $\cos n\phi + i\sin n\phi$

(c) $\sin n\phi + i\cos n\phi$ (d) $\sin n\phi - i\cos n\phi$

**81. If *n* is a positive integer, then** $(1+i)^n + (1-i)^n$ **is equal to**

(a) $(\sqrt{2})^{n-2} \cos\left(\frac{n\pi}{4}\right)$ (b) $(\sqrt{2})^{n-2} \sin\left(\frac{n\pi}{4}\right)$

(c) $(\sqrt{2})^{n+2} \cos\left(\frac{n\pi}{4}\right)$ (d) $(\sqrt{2})^{n+2} \sin\left(\frac{n\pi}{4}\right)$

**82.** $\left[\frac{1+\cos(\pi/8)+i\sin(\pi/8)}{1+\cos(\pi/8)-i\sin(\pi/8)}\right]^8$ **is equal to**

(a) – 1 (b) 0

(c) 1 (d) 2

**83. If** $\omega$ **is a cube root of unity, then** $(1+\omega)^3 - (1+\omega^2)^3 =$

(a) 0 (b) $\omega$

(c) $\omega^2$ (d) None of these

**84. If** $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)........ (\cos n\theta + i\sin n\theta) = 1$ **, then the value of** $\theta$ **is**

(a) $4m\pi$ (b) $\frac{2m\pi}{n(n+1)}$

(c) $\frac{4m\pi}{n(n+1)}$ (d) $\frac{m\pi}{n(n+1)}$

**85.** $(-\sqrt{3}+i)^{53}$ **where** $i^2 = -1$ **is equal to**

(a) $2^{53}(\sqrt{3}+2i)$ (b) $2^{52}(\sqrt{3}-i)$

(c) $2^{53}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$ (d) $2^{53}(\sqrt{3}-i)$

**86.** $\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^5} =$

(a) $\cos(4\alpha+5\beta)+i\sin(4\alpha+5\beta)$

(b) $\cos(4\alpha+5\beta)-i\sin(4\alpha+5\beta)$

(c) $\sin(4\alpha+5\beta)-i\cos(4\alpha+5\beta)$

(d) None of these

**87.** $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n =$

(a) $\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$ (b) $\cos\left(\frac{n\pi}{2}+n\theta\right)+i\sin\left(\frac{n\pi}{2}+n\theta\right)$

(c) $\sin\left(\frac{n\pi}{2}-n\theta\right)+i\cos\left(\frac{n\pi}{2}-n\theta\right)$ (d) $\cos n\left(\frac{\pi}{2}+2\theta\right)+i\sin n\left(\frac{\pi}{2}+2\theta\right)$

**88. The product of all the roots of** $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ **is**

(a) $-1$ (b) 1

(c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

**89. If** $\frac{1}{x}+x=2\cos\theta,$ **then** $x^n+\frac{1}{x^n}$ **is equal to**

(a) $2\cos n\theta$ (b) $2\sin n\theta$

(c) $\cos n\theta$ (d) $\sin n\theta$

**90. The value of** $\left[\frac{1-\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}}{1-\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}}\right]^{10}=$

(a) 0 (b) – 1

(c) 1 (d) 2

**91. If** $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ **then** $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$ **equals**

(a) $2\cos(\alpha+\beta+\gamma)$ (b) $\cos 2(\alpha+\beta+\gamma)$

(c) 0 (d) 1

**92. If** $\omega$ **is a cube root of unity, then** $(1+\omega-\omega^2)(1-\omega+\omega^2)$

(a) 1 (b) 0

(c) 2 (d) 4

**93. If** $\omega$ **is a cube root of unity, then the value of** $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5=$

(a) 16 (b) 32

(c) 48 (d) – 32

**94. If** $\omega$ **is a complex cube root of unity, then** $(x-y)(x\omega-y)(x\omega^2-y)=$

(a) $x^2+y^2$

(b) $x^2-y^2$

(c) $x^3-y^3$

(d) $x^3+y^3$

**95. The value of** $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$ **will be**

(a) 1 (b) – 1

(c) 2 (d) – 2

**96. If** $z_1, z_2, z_3......n_n$ **are** ***n*** **th, roots of unity, then for** $k=1,2,......,n$

(a) $|z_k|=k|z_{k+1}|$ (b) $|z_{k+1}|=k|z_k|$

(c) $|z_{k+1}|=|z_k|+|z_{k+1}|$ (d) $|z_k|\neq|z_{k+1}|$

**97.** $\frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}}+\frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}$ **is equal to**

(a) – 64 (b) – 32

(c) – 16 (d) $\frac{1}{16}$

**98.** $\left(\frac{-1+i\sqrt{3}}{2}\right)^{20}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{20}=$

(a) $20\sqrt{3}i$ (b) 1

(c) $\frac{1}{2^{19}}$ (d) $-1$

**99. If** $z+z^{-1}=1$, then $z^{100}+z^{-100}$ **is equal to**

(a) *i* (b) – *I* (c) 1 (d)– 1

**100. If** $\frac{1+\sqrt{3}\,i}{2}$ **is a root of equation** $x^4-x^3+x-1=0$ **then its real roots are**

(a) 1, 1 (b) – 1, – 1

(c) 1, – 1 (d) 1, 2

**101. If** $\omega$ **is a complex cube root of unity, then**

$225+(3\omega+8\omega^2)^2+(3\omega^2+8\omega)^2=$

(a) 72 (b) 192

(c) 200 (d) 248

**102.** $\sinh ix$ is

(a) $i\sin(ix)$ (b) $i\sin x$

(c) $-i\sin x$ (d) $\sin(ix)$

**103. If $1, \omega, \omega^2$ are the cube roots of unity, then**

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix} =$$

(a) 0 (b) 1

(c) $\omega$ (d) $\omega^2$

# COMPLEX NUMBERS AND DE MOIVRE'S THEOREM

## HINTS AND SOLUTIONS

1. (c) $\left(\frac{1+i}{1-i}\right)^2+\left(\frac{1-i}{1+i}\right)^2=\frac{2i}{-2i}+\left(\frac{-2i}{2i}\right)=-2$

2. (d) $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

$=\left[\frac{1+2i}{1^2+2^2}+\frac{3-3i}{1^2+1^2}\right]\left[\frac{6-16+12i+8i}{2^2+4^2}\right]$

**3.** (c) We have $\frac{(1+i)^2}{2-i}=\frac{(2i)(2+i)}{(2-i)(2+i)}=-\frac{2}{5}+i\frac{4}{5}$.

4. (b) $\frac{1+i}{1-i}=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{(1+i)^2}{2}=\frac{2i}{2}=i$

$\therefore\left(\frac{1+i}{1-i}\right)^m=i^m=1$

5. (c) Let $z=i^{[1+3+5+....+(2n+1)]}$

Clearly series is A.P. with common difference = 2

$\because T_n=2n-1$ and $T_{n+1}=2n+1$

6. (c) $x+iy=\frac{4+i}{2-3i}=\frac{(4+i)(2+3i)}{13}=\frac{5}{13}+\frac{14}{13}i$

7. (c) $\frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)}=\left(\frac{3-4\sin^2\theta}{1+4\sin^2\theta}\right)+i\left(\frac{8\sin\theta}{1+4\sin^2\theta}\right)$

Now, since it is real, therefore Im$(z)=0$

$\Rightarrow\frac{8\sin\theta}{1+4\sin^2\theta}=0\Rightarrow\sin\theta=0$, $\therefore\theta=n\pi$

8. (c) $a=\cos\theta+i\sin\theta.$

$\therefore\frac{1+a}{1-a}=\frac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta}.$

Rationalization of denominator, we get $\frac{1+a}{1-a}=\frac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta}\times\frac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta)+i\sin\theta}$

9. (b) Verification

10 (d) If $z_1=1-i$ and $z_2=-2+4i$

Then $\frac{z_1z_2}{z_1}=\frac{(1-i)(-2+4i)}{1-i}=-2+4i\Rightarrow\text{Im}\left(\frac{z_1z_2}{z_1}\right)=4$.

**11.** (b) $(x+iy)^{1/3} = a+ib \Rightarrow (x+iy) = (a+ib)^3$

$= a^3 + 3a^2.ib + 3a.(ib)^2 + (ib)^3$

$= a^3 - 3ab^2 + i(3a^2b - b^3)$

Equating real and imaginary parts, we get

$\frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$

$\therefore \frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

**12.** (b) $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$

$\Rightarrow (4+2i)x + (9-7i)y - 3i - 3 = 10i$

Equating real and imaginary parts, we get $2x - 7y = 13$ and $4x + 9y = 3$. Hence $x = 3$ and $y = -1$.

**13.** (c) $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely imaginary, if the real part vanishes, *i.e.*, $\frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow 3 - 4\sin^2\theta = 0$

**14.** (b) If $(1-i)^n = 2^n$ ......(i)

We know that if two complex numbers are equal, their moduli must also be equal, therefore from (i), we have

$|(1-i)^n| = |2^n| \Rightarrow |1-i|^n = |2|^n$, $(\because 2^n > 0)$

$\Rightarrow \left[\sqrt{1^2 + (-1)^2}\right]^n = 2^n \Rightarrow (\sqrt{2})^n = 2^n$

$\Rightarrow 2^{n/2} = 2^n \Rightarrow \frac{n}{2} = n \Rightarrow n = 0$

**15.** (b) We have $(1+i)^{2n} = (1-i)^{2n}$

$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1 \Rightarrow (i)^{2n} = 1 \Rightarrow (i)^{2n} = (-1)^2$

$\Rightarrow (i)^{2n} = (i^2)^2 \Rightarrow (i)^{2n} = (i)^4 \Rightarrow 2n = 4 \Rightarrow n = 2$.

**16.** (d) $\{(1-\cos\theta) + i.2\sin\theta\}^{-1} = \left\{2\sin^2\frac{\theta}{2} + i.4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}^{-1}$

$= \left(2\sin\frac{\theta}{2}\right)^{-1}\left\{\sin\frac{\theta}{2} + i.2\cos\frac{\theta}{2}\right\}^{-1}$

$= \left(2\sin\frac{\theta}{2}\right)^{-1} \frac{1}{\sin\frac{\theta}{2} + i.2\cos\frac{\theta}{2}} \times \frac{\sin\frac{\theta}{2} - i.2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2} - i.2\cos\frac{\theta}{2}}$

**17.** (c) Given $z = 1 + i$ and $i = \sqrt{-1}$. Squaring both sides, we get $z^2 = (1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1$ or $z^2 = 2i.$

Since it is multiplicative identity, therefore multiplicative inverse of $z^2 = \frac{1}{2i} \times \frac{i}{i} = \frac{i}{2i^2} = -\frac{i}{2}.$

**18.** (b) If $x + iy = \frac{3}{2 + \cos\theta + i\sin\theta}$

$$= \frac{3(2 + \cos\theta - i\sin\theta)}{(2+\cos\theta)^2 + \sin^2\theta} = \frac{6 + 3\cos\theta - 3i\sin\theta}{4 + \cos^2\theta + 4\cos\theta + \sin^2\theta}$$

$$= \left[\frac{6 + 3\cos\theta}{5 + 4\cos\theta}\right] + i\left[\frac{-3\sin\theta}{5 + 4\cos\theta}\right]$$

**19.** (a) $\frac{1+iz}{1-iz} = \frac{1 + i(b+ic)/(1+a)}{1 - i(b+ic)/(1+a)} = \frac{1+a-c+ib}{1+a+c-ib}$

$$= \frac{(1+a-c+ib)(1+a+c+ib)}{(1+a+c)^2 + b^2}$$

**20.** (c) $(x+iy)(p+iq) = (x^2+y^2)i$

$$\Rightarrow (xp - yq) + i(xq + yp) = (x^2 + y^2)i$$

$$\Rightarrow xp - yq = 0, xq + yp = x^2 + y^2$$

$$\Rightarrow \frac{x}{q} = \frac{y}{p} \text{ and } xq + yp = x^2 + y^2$$

21. (d) $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy \quad = 0$

22. (b) Given, $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$ ; $\left[\left(\frac{1-i}{1+i}\right) \times \left(\frac{1-i}{1-i}\right)\right] = a + ib$

$$\Rightarrow a + ib = \left[\frac{(1-i)^2}{2}\right]^{100} = \left[\frac{-2i}{2}\right]^{100} = (-i)^{100}$$

$$\Rightarrow a + ib = \left[(i)^4\right]^{25} = 1 + 0i,$$

**23.** (c) Given that $a^2 + b^2 = 1$, therefore

$$\frac{1+b+ia}{1+b-ia} = \frac{(1+b+ia)(1+b+ia)}{(1+b-ia)(1+b+ia)}$$

$$= \frac{(1+b)^2 - a^2 + 2ia(1+b)}{1 + b^2 + 2b + a^2} = \frac{(1-a^2) + 2b + b^2 + 2ia(1+b)}{2(1+b)}$$

$$= \frac{2b^2 + 2b + 2ia(1+b)}{2(1+b)} = b + ia$$

**24.** (a) Given that $z = 3 - 4i \Rightarrow z^2 = -7 - 24i$,

$z^4 = -117 - 44i$ and $z^4 = -527 + 336i$

$\therefore z^4 - 3z^3 + 3z^2 + 99z - 95 = 5$

**25.** (c) $z = \frac{(2+i)^2}{3+i} = \frac{3+4i}{3+i} \times \frac{3-i}{3-i} = \frac{13}{10} + i\frac{9}{10}$

Conjugate $= \frac{13}{10} - i\frac{9}{10}$.

**26.** (a) Assume any two complex numbers satisfying both conditions *i.e.*, $z_1 \neq z_2$ and $|z_1| = |z_2|$

Let $z_1 = 2 + i, z_2 = 1 - 2i, \therefore \frac{z_1 + z_2}{z_1 - z_2} = \frac{3-i}{1+3i} = -i$

Hence the result.

**27.** (b) $\left|z + \frac{2}{z}\right| = 2 \Rightarrow |z| - \frac{2}{|z|} \leq 2 \Rightarrow |z|^2 - 2|z| - 2 \leq 0$

$|z| \leq \frac{2 \pm \sqrt{4+8}}{2} \leq 1 \pm \sqrt{3}$.

Hence max. value of $|z|$ is $1 + \sqrt{3}$

**28.** (a) Let $z = x + iy, \bar{z} = x - iy$ and $z^{-1} = \frac{1}{x+iy}$

$\Rightarrow \overline{(z^{-1})} = \frac{x+iy}{x^2+y^2}$; $\therefore \overline{(z^{-1})}\bar{z} = \frac{x+iy}{x^2+y^2}(x - iy) = 1$

**29.** (c) $z = 3 + 5i$, $\bar{z} = 3 - 5i$

$\Rightarrow z^3 = (3+5i)^3 = 3^3 + (5i)^3 + 3.3.5i(3+5i)$

$= -198 + 10i$

Hence, $z^3 + \bar{z} + 198 = 10i - 198 + 3 - 5i + 198 = 3 + 5i$.

**30.** (d) Let $z = x + iy$, then

**31.** (d) Check by putting $z_1 = 1 + 0i$ and $z_2 = 0 + i$

**32.** (d) $|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1, z_2$ lies on same straight line.

$\therefore arg\, z_1 = arg\, z_2 \Rightarrow arg\, z_1 - arg\, z_2 = 0$

**33.** (d) $amp(z) = \tan^{-1}\frac{\sin\alpha}{1 - \cos\alpha} \quad = \tan^{-1}\left(\cot\frac{\alpha}{2}\right) = \tan^{-1}\left\{\tan\left(\frac{\pi}{2} - \frac{\alpha}{2}\right)\right\} = \frac{\pi}{2} - \frac{\alpha}{2}$.

**34.** (b) $z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{i}{2}$

$\therefore |z| = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

and $arg(z) = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1/2}{\sqrt{3}/2}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow arg(z) = \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$.

**35.** (b) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

Then $|z_1| = |z_2| \Rightarrow |z_2| = r_1$

and $arg(z_1) + arg(z_2) = 0 \Rightarrow arg(z_2) = -arg(z_1) = -\theta_1$

$z_2 = r_1[\cos(-\theta_1) - i\sin(-\theta_1)] = r_1(\cos\theta_1 - i\sin\theta_1)$

$= \bar{z}_1 \; \bar{z}_1 = z_2$.

**36.** (b) concept.

**37.** (d) Let $z = x + iy, \bar{z} = x - iy$

Since $arg(z) = \theta = \tan^{-1}\frac{y}{x}$

$arg(\bar{z}) = \theta = \tan^{-1}\left(\frac{-y}{x}\right)$

Thus $arg(z) \neq arg(\bar{z})$.

**38.** (c) We have $|z_k| = 1, k = 1, 2, \ldots. n$

$\Rightarrow |z_k|^2 = 1 \Rightarrow z_k \bar{z}_k = 1 \Rightarrow \bar{z}_k = \frac{1}{z_k}$

Therefore $|z_1 + z_2 + \ldots. + z_n| = |\overline{z_1 + z_2 + \ldots. + z_n}|$

$(\because |z| = |\bar{z}|)$

$= |\bar{z}_1 + \bar{z}_2 + \ldots.. + \bar{z}_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots. + \frac{1}{z_n}\right|$

**39.** (b) $\frac{2-3i}{4-i} = \frac{(2-3i)(4+i)}{(4+i)(4-i)} = \frac{8+3-12i+2i}{16+1} = \frac{11-10i}{17}$

$\Rightarrow$Conjugate $= \frac{11+10i}{17}$.

**40.** (c) Given $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1 \Rightarrow \frac{z_1+z_2}{z_1-z_2} = \cos\theta + i\sin\theta$ (say)

$\Rightarrow \frac{z_1}{z_2} = \frac{1+\cos\theta + i\sin\theta}{-1+\cos\theta + i\sin\theta} = -i\cot\frac{\theta}{2}$

Which is zero, if $\theta = n\pi (n \in I)$, and is otherwise purely imaginary.

**41.** (c) $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1 \Rightarrow \frac{z_1 - z_2}{z_1 + z_2} = \cos\alpha + i\sin\alpha$

$\Rightarrow \frac{2z_1}{-2z_2} = \frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha - 1 + i\sin\alpha}$ (Applying componendo and dividendo)

**42.** (a) $|z| = 1 \Rightarrow |x + iy| = 1 \Rightarrow x^2 + y^2 = 1$

$$\omega = \frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x^2+y^2-1)}{(x+1)^2+y^2} + \frac{2iy}{(x+1)^2+y^2} = \frac{2iy}{(x+1)^2+y^2}$$

**43.** (b)

**44.** (b) We have $|z_1| = 1$ and $z_2$ be any complex number.

$$\Rightarrow \left|\frac{z_1 - z_2}{1 - z_1\bar{z}_2}\right| = \frac{|z_1 - z_2|}{\left|1 - \frac{\bar{z}_2}{\bar{z}_1}\right|}; \qquad \because z_1\bar{z}_1 = |z_1|^2$$

$= \frac{|z_1 - z_2|}{|\bar{z}_1 - \bar{z}_2|}|\bar{z}_1|$; Given that $\because |\bar{z}_1| = 1$

$$= \frac{|z_1 - z_2|}{|\overline{z_1 - z_2}|} = \frac{|z_1 - z_2|}{|z_1 - z_2|} = 1.$$

**45.** (a) We have $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2) = |z_1|^2 + |z_2|^2$

Where $\theta_1 = arg(z_1), \theta_2 = arg(z_2)$

$\Rightarrow \cos(\theta_1 - \theta_2) = 0 \Rightarrow \theta_1 - \theta_2 = \frac{\pi}{2}$

$\Rightarrow arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2} \Rightarrow \text{Re}\left(\frac{z_1}{z_2}\right) = \frac{|z_1|}{|z_2|}\cos\left(\frac{\pi}{2}\right) = 0$

**46.** (c) Squaring the given relations implies that

$x_1x_2 + y_1y_2 = 0$

Now $amp\ z_1 - amp\ z_2 = \tan^{-1}\frac{y_1}{x_1} - \tan^{-1}\frac{y_2}{x_2}$

$$= \tan^{-1}\frac{\frac{y_1}{x_1} - \frac{y_2}{x_2}}{1 + \frac{y_1y_2}{x_1x_2}} = \tan^{-1}\frac{y_1x_2 - y_2x_1}{x_1x_2 + y_1y_2} = \tan^{-1}\infty = \frac{\pi}{2}.$$

**47.** (d) $|z||\omega|=1$ ......(i)

and $arg\left(\frac{z}{\omega}\right)=\frac{\pi}{2} \Rightarrow \frac{z}{\omega}=i \Rightarrow \left|\frac{z}{\omega}\right|=1$ .....(ii)

From equation (i) and (ii)

$|z|=|\omega|=1$ and $\frac{z}{\omega}+\frac{\bar{z}}{\bar{\omega}}=0;\ z\bar{\omega}+\bar{z}\omega=0$

$\bar{z}\omega=-z\bar{\omega}=\frac{-z}{\omega}\bar{\omega}\omega$ ; $\bar{z}\omega=-i|\omega|^2=-i.$ .

**48.** (c) $arg\left(\frac{3+i}{2-i}+\frac{3-i}{2+i}\right)=arg\left(\frac{6+5i+i^2+6-5i+i^2}{5}\right)$

$=arg\left(\frac{10}{5}\right)=0$ .

**49.** (a) We know that the principal value of $\theta$ lies between $-\pi$ and $\pi$.

**50.** (a) Concept

**51.** (d) Given $z_1=1+2i$, $z_2=3+5i$ and $\bar{z}_2=3-5i$

$\frac{\bar{z}_2 z_1}{z_2}=\frac{(3-5i)(1+2i)}{(3+5i)}=\frac{13+i}{3+5i}$

$=\frac{13+i}{3+5i}\times\frac{3-5i}{3-5i}=\frac{44-62i}{34}$

Then $\text{Re}\left(\frac{\bar{z}_2 z_1}{z_2}\right)=\frac{44}{34}=\frac{22}{17}$.

**52.** (c) $(\sqrt{8}+i)^{50}=3^{49}(a+ib)$

Taking modulus and squaring on both sides, we get

$(8+1)^{50}=3^{98}(a^2+b^2)$

$9^{50}=3^{98}(a^2+b^2)$

$3^{100}=3^{98}(a^2+b^2)$

$\Rightarrow (a^2+b^2)=9$ .

**53.** (a) We have $arg\left(\frac{z_1}{z_2}\right)=\pi$

$\Rightarrow arg(z_1)-arg(z_2)=\pi \Rightarrow\ arg\,(z_1)=arg\,(z_2)+\pi$

Let $arg\,(z_2)=\theta$, then $arg\,(z_1)=\pi+\theta$

$\therefore z_1=|z_1|\,[\cos(\pi+\theta)+i\sin(\pi+\theta)]$

$= |z_1| (-\cos\theta - i\sin\theta)$

and $z_2 = |z_2| (\cos\theta + i\sin\theta) = |z_1| (\cos\theta + i\sin\theta)$

$(\because |z_1| = |z_2|)$

Hence $z_1 + z_2 = 0$.

**54.** (a) Let $z = (1-i)^{-i}$. Taking log on both sides,

$\Rightarrow \log z = -i\log(1-i) = -i\log\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$

$= -i\log\left(\sqrt{2}e^{-i\pi/4}\right) = -i\left[\frac{1}{2}\log 2 + \log e^{-i\pi/4}\right]$

$= -i\left[\frac{1}{2}\log 2 - \frac{i\pi}{4}\right] = -\frac{i}{2}\log 2 - \frac{\pi}{4}$

$\Rightarrow z = e^{-\pi/4}\, e^{-i/2\log 2}$. Taking real part only

**55.** (d) $\sqrt{a+ib} = x+yi \Rightarrow \left(\sqrt{a+ib}\right)^2 = (x+yi)^2$

$\Rightarrow a = x^2 - y^2, b = 2xy$ and hence

$\sqrt{a-ib} = \sqrt{x^2 - y^2 - 2xyi} = \sqrt{(x-yi)^2} = x - iy$

**56.** (b) Let $z = i\log\left(\frac{x-i}{x+i}\right) \Rightarrow \frac{z}{i} = \log\left(\frac{x-i}{x+i}\right)$

$\Rightarrow \frac{z}{i} = \log\left[\frac{x-i}{x+i}\times\frac{x-i}{x-i}\right] = \log\left[\frac{x^2-1-2ix}{x^2+1}\right]$

$\Rightarrow \frac{z}{i} = \log\left[\frac{x^2-1}{x^2+1} - i\frac{2x}{x^2+1}\right]$ ......(i)

$\because \log(a+ib) = \log(re^{i\theta}) = \log r + i\theta$

$= \log\sqrt{a^2+b^2} + i\tan^{-1}(b/a)$

Hence, $\frac{z}{i} = \log\sqrt{\left(\frac{x^2-1}{x^2+1}\right)^2 + \left(\frac{-2x}{x^2+1}\right)^2} + i\tan^{-1}\left(\frac{-2x}{x^2-1}\right)$

[by $eq^n$. (i)]

$\frac{z}{i} = \log\frac{\sqrt{x^4+1-2x^2+4x^2}}{(x^2+1)^2} + i\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \log 1 + i(2\tan^{-1}x) = 0 + i(2\tan^{-1}x)$

$\therefore z = i^2\, 2\tan^{-1}x = -2\tan^{-1}x = \pi - 2\tan^{-1}x$.

**57.** (a) $\frac{1+7i}{(2-i)^2} = \frac{(1+7i)}{(3-4i)}\frac{(3+4i)}{(3+4i)} = \frac{-25+25i}{25} = -1+i$

Let $z = x+iy = -1+i$

$\therefore r\cos\theta = -1$ and $r\sin\theta = 1$ $\therefore \theta = \frac{3\pi}{4}$ and $r = \sqrt{2}$

Thus $\frac{1+7i}{(2-i)^2} = \sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$

**58.** (c) Since $\frac{-\sqrt{3}-i}{2} = -\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\Rightarrow \left(\frac{-\sqrt{3}-i}{2}\right)^3 = -\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3 = -i$

and $\frac{\sqrt{3}-i}{2} = \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$

and $\left(\frac{\sqrt{3}-i}{2}\right)^3 = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = -i$.

Hence the result.

**59.** (a) $x + iy = \sqrt{\frac{a+ib}{c+id}} \Rightarrow x - iy = \sqrt{\frac{a-ib}{c-id}}$

Also $x^2 + y^2 = (x+iy)(x-iy) = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$

$\Rightarrow (x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

**60.** (c) $1 + i\sqrt{3} = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right] = 2e^{i\pi/3}$

$\therefore (1+i\sqrt{3})^9 = (2e^{i\pi/3})^9 = 2^9.e^{i(3\pi)}$

$= 2^9(\cos 3\pi + i\sin 3\pi) = -2^9$

$\therefore a + ib = (1+i\sqrt{3})^9 = -2^9$; $\therefore b = 0$.

**61.** (d) Since $1 - i = \sqrt{2}\left\{\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right\}, |1-i| = \sqrt{2}$

$\therefore |1-i|^x = 2^x \Rightarrow (\sqrt{2})^x = 2^x \Rightarrow 2^{x/2} = 2^x$

$\Rightarrow \frac{x}{2} = x \Rightarrow x = 0$

**62.**(c) $\tan^{-1}\left(\frac{5i}{3}\right) = i\tan^{-1}\left(\frac{5}{3}\right) = \frac{i}{2}\log\left(\frac{\frac{5}{3}+1}{\frac{5}{3}-1}\right)$

$\mathrm{Im}\left(\tan^{-1}\left(\frac{5i}{3}\right)\right) = \frac{1}{2}\log 4 = \frac{1}{2}.2\log 2 = \log 2$.

**63.** (a) $y=\cos\theta+i\sin\theta=e^{i\theta}$, then $\frac{1}{y}=e^{-i\theta}=\cos\theta-i\sin\theta$

$\therefore y+\frac{1}{y}=2\cos\theta$.

**64.** (b) $\frac{1-i}{1+i}=\frac{(1-i)(1-i)}{(1+i)(1-i)}=\frac{1+(i)^2-2i}{1+1}=-i$

which can be written as $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$

**65.** (b) We have $|z^2-1|=|z|^2+1$

$\Rightarrow |(x+iy)^2-1|=|x+iy|^2+1$

$\Rightarrow |(x^2-y^2-1)+2xyi|=\left(\sqrt{x^2+y^2}\right)^2+1$

**66.** (b) By adding $a\bar{a}$ on both the sides of $z\bar{z}+a\bar{z}+\bar{a}z=-b$

we get, $(z+a)(\bar{z}+\bar{a})=a\bar{a}-b$

$\Rightarrow |z+a|^2=|a|^2-b, \{\because z\bar{z}=|z|^2\}$

This equation will represent a circle with centre $z=-a$, if $|a|^2-b>0, i.e. |a|^2>b$ since $|a|^2=b$ represents point circle only.

**67.** (c) We have $z_k=1+a+a^2+.....+a^{k-1}=\frac{1-a^k}{1-a}$

$\Rightarrow z_k-\frac{1}{1-a}=\frac{-a^k}{1-a}$

$\Rightarrow \left|z_k-\frac{1}{1-a}\right|=\frac{|a^k|}{|1-a|}=\frac{|a|^k}{|1-a|}<\frac{1}{|1-a|}$

$\Rightarrow z_k$ lies within $\left|z-\frac{1}{1-a}\right|=\frac{1}{|1-a|}$.

**68.** (c) Let $r$ be the circum radius of the equilateral triangle and $\omega$ the cube root of unity.

**69.** (d)standard problem

**70.** (d) $z_2=z_1e^{2i\pi/3}=\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

$=\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)=-\frac{3}{4}-\frac{1}{4}=-1$.

**71.** (b) $|z-2|+|z+2|=8$

$\Rightarrow \sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=8$

$\Rightarrow x^2 + y^2 + 4 - 4x = 64 + x^2 + y^2 + 4 + 4x - 16\sqrt{(x+2)^2 + y^2}$

$\Rightarrow -8x - 64 = -16\sqrt{(x+2)^2 + y^2}$

$\Rightarrow (x+8) = 2\sqrt{(x+2)^2 + y^2}$

$\Rightarrow x^2 + 64 + 16x = 4[x^2 + y^2 + 4 + 4x]$

$\Rightarrow 3x^2 + 4y^2 - 48 = 0 \Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$,

Which is an ellipse.

**72.** (a) We have $|z - z_1| = |z - z_2| = |z - z_3| = |z - z_4|$

Therefore the point having affix $z$ is equidistant from the four points having affixes $z_1, z_2, z_3, z_4$. Thus $z$ is the affix of either the centre of a circle or the point of intersection of diagonals of a square or rectangle. Therefore $z_1, z_2, z_3, z_4$ are either concyclic or vertices of a square. Hence $z_1, z_2, z_3, z_4$ are concyclic.

**73.** (b) Let $z = x + iy$; $z + iz = (x - y) + i(x + y)$ and $iz = -y + ix$

If $A$ denotes the area of the triangle formed by $z, z + iz$ and $iz$, then $A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ x - y & x + y & 1 \\ -y & x & 1 \end{vmatrix}$

Applying transformation $R_2 \to R_2 - R_1 - R_3$, we get

$$A = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 0 & 0 & -1 \\ -y & x & 0 \end{vmatrix} = \frac{1}{2}(x^2 + y^2) = \frac{1}{2}|z|^2$$

**74.** (b) $|z + 1| = \sqrt{2}|z - 1|$

Putting $z = x + iy \Rightarrow |x + iy + 1| = \sqrt{2}|x + iy - 1|$

$\Rightarrow |(x+1) + iy| = \sqrt{2}|(x-1) + iy|$

$\Rightarrow (x+1)^2 + y^2 = 2[(x-1)^2 + y^2]$

$\Rightarrow x^2 + y^2 - 6x + 1 = 0$.

Which is the equation of a circle.

**75.** (a) Area of the triangle $\frac{1}{2}|z|^2 = 18 \Rightarrow |z| = 6$.

**76.** (c) We have $|z - 1| + |z + 1| \le 4$

$\Rightarrow |z - 1|^2 + |z + 1|^2 + 2|z - 1||z + 1| \le 16$

$\Rightarrow (z - 1)(\bar{z} - 1) + (z + 1)(\bar{z} + 1) + 2|(z - 1)(z + 1)| \le 16$

$\Rightarrow 2|z|^2 + 2 + 2|z^2 - 1| \le 16 \Rightarrow |z|^2 + |z^2 - 1| \le 7$

**77.** (c) We have $\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1 \Rightarrow z_1^2 + z_2^2 = z_1 z_2$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_1 z_3 + z_2 z_3$, where $z_3 = 0$

$\Rightarrow z_1, z_2$ and the origin $(\because z_3 = 0)$ form an equilateral triangle.

**78.** (c) $|z - 1| = |z + i| \Rightarrow |x - 1 + iy|^2 = |x + i(y + 1)|^2$

$\Rightarrow (x - 1)^2 + y^2 = x^2 + (y + 1)^2$

$\Rightarrow x + y = 0$ *i.e.*, a straight line through the origin.

**79.** (a) $|z - 2 + i| = |z - 3 - i|$

$\Rightarrow |(x - 2) + i(y + 1)| = |(x - 3) + i(y - 1)|$

$\Rightarrow \sqrt{(x - 2)^2 + (y + 1)^2} = \sqrt{(x - 3)^2 + (y - 1)^2}$

$\Rightarrow x^2 + 4 - 4x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 1 - 2y$

$\Rightarrow 2x + 4y - 5 = 0$.

**80.** (b) L.H.S. $= \left[\frac{2\cos^2(\phi/2) + 2i\sin(\phi/2)\cos(\phi/2)}{2\cos^2(\phi/2) - 2i\sin(\phi/2)\cos(\phi/2)}\right]^n$

$= \left[\frac{\cos(\phi/2) + i\sin(\phi/2)}{\cos(\phi/2) - i\sin(\phi/2)}\right]^n = \left[\frac{e^{i(\phi/2)}}{e^{-i(\phi/2)}}\right]^n = (e^{i\phi})^n$

$= \cos n\phi + i\sin n\phi$.

**81.** (c) $(1 + i)^n + (1 - i)^n$

$= (2)^{n/2}\left\{\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4} + \cos\frac{n\pi}{4} - i\sin\frac{n\pi}{4}\right\}$

$= 2^{\frac{n}{2}}.2\cos\frac{n\pi}{4} = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4} = (\sqrt{2})^{n+2}\cos\frac{n\pi}{4}$.

**82.** (a) $\left[\frac{1 + \cos(\pi/8) + i\sin(\pi/8)}{1 + \cos(\pi/8) - i\sin(\pi/8)}\right]^8$

$= \left[\frac{2\cos^2(\pi/16) + 2i\sin(\pi/16)\cos(\pi/16)}{2\cos^2(\pi/16) - 2i\sin(\pi/16)\cos(\pi/16)}\right]^8$

$= \frac{[\cos(\pi/16) + i\sin(\pi/16)]^8}{[\cos(\pi/16) - i\sin(\pi/16)]^8}$

$= \left[\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right]^8\left[\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right]^8$

$= [\cos(\pi/16) + i\sin(\pi/16)]^{16}$

$= \cos 16\left(\frac{\pi}{16}\right) + i\sin 16\left(\frac{\pi}{16}\right) = \cos\pi = -1$.

**83.** (a) $(1+\omega)^3-(1+\omega^2)^3=(-\omega^2)^3-(-\omega)^3$

$$=-\omega^6+\omega^3=-\omega^3\omega^3+\omega^3=-1+1=0$$

**84.** (c) We have $(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)$

$\ldots\ldots(\cos n\theta+i\sin n\theta)=1$

$\Rightarrow \cos(\theta+2\theta+3\theta+\ldots+n\theta)+i\sin(\theta+2\theta+.+n\theta)=1$

$\Rightarrow \cos\left(\frac{n(n+1)}{2}\theta\right)+i\sin\left(\frac{n(n+1)}{2}\theta\right)=1$

$\cos\left(\frac{n(n+1)}{2}\theta\right)=1$ and $\sin\left(\frac{n(n+1)}{2}\theta\right)=0$

**85.** (c) $(-\sqrt{3}+i)^{53}=2^{53}\left(\frac{-\sqrt{3}}{2}+\frac{i}{2}\right)^{53}$

$=2^{53}(\cos 150^o+i\sin 150^o)^{53}$

$=2^{53}[\cos(150^o\times 53)+i\sin(150^o\times 53)]$

$=2^{53}[\cos(22\pi+30^o)+i\sin(22\pi+30^o)]$

$=2^{53}[\cos 30^o+i\sin 30^o]=2^{53}\left[\frac{\sqrt{3}}{2}+i\frac{1}{2}\right]$.

**86.** (c) $\frac{(\cos\alpha+i\sin\alpha)^4}{(\sin\beta+i\cos\beta)^5}=\frac{\cos 4\alpha+i\sin 4\alpha}{i^5(\cos\beta-i\sin\beta)^5}$

$=-i(\cos 4\alpha+i\sin 4\alpha)(\cos\beta-i\sin\beta)^{-5}$

$=-i[\cos 4\alpha+i\sin 4\alpha]\,[\cos 5\beta+i\sin 5\beta]$

$=-i[\cos(4\alpha+5\beta)+i\sin(4\alpha+5\beta)]$

**87.** (a) $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n=\left(\frac{1+\cos\alpha+i\sin\alpha}{1+\cos\alpha-i\sin\alpha}\right)^n$

$$=\left(\frac{2\cos^2\frac{\alpha}{2}+2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}-2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right)^n=\left(\frac{\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-i\sin\frac{\alpha}{2}}\right)^n$$

**88.** (b) Given that $\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right]^{3/4}=[\cos\pi+i\sin\pi]^{1/4}$.

Since the expression has only 4 different roots, therefore on putting $n=0,1,2,3$ in $\cos\left[\frac{2n\pi+\pi}{4}\right]+i\sin\left[\frac{2n\pi+\pi}{4}\right]$ and multiplying them,

We get $=\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]\left[\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right]\left[\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right]\left[\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right]$

**89.** (a) $x+\frac{1}{x}=2\cos\theta$

$\Rightarrow x^2-2x\cos\theta+1=0 \Rightarrow \quad x=\cos\theta\pm i\sin\theta$

$\Rightarrow x^n=\cos n\theta\pm i\sin n\theta \Rightarrow \frac{1}{x}=\frac{1}{\cos\theta\pm i\sin\theta}$

$\Rightarrow \frac{1}{x}=\cos\theta\mp i\sin\theta \Rightarrow \frac{1}{x^n}=\cos n\theta\mp i\sin n\theta$

**90.** (b) Let $\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}=z$ and $\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}=\frac{1}{z}$

Therefore, $\left(\frac{1-z}{1-\frac{1}{z}}\right)^{10}=\left\{\frac{-(z-1)z}{(z-1)}\right\}^{10}=(-z)^{10}$

**91.** (c) standard problem

**92.** (d) If $\omega$ is a complex cube root of unity then $\omega^3=1$ and $1+\omega+\omega^2=0$, therefore $(1+\omega-\omega^2)(1-\omega+\omega^2)$

$=(-2\omega^2)(-2\omega)=4$

**93.** (b) $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$

$=(-2\omega)^5+(-2\omega^2)^5=-32\omega^3\omega^2-32(\omega^3)^3\omega$

$=-32(\omega^2+\omega)=-32(-1)=32$

94. (c) $(x-y)(x\omega-y)(x\omega^2-y)$

$=(x^2\omega-xy-xy\omega+y^2)(x\omega^2-y)$

$=x^3-x^2y(1+\omega+\omega^2)+xy^2(1+\omega+\omega^2)-y^3$

$=x^3-y^3$

**95.** (b) Multiplying the numerator and denominator by $\omega$ and $\omega^2$ respectively I and II expressions

$=\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=\frac{\omega(a+b\omega+c\omega^2)}{(b\omega+c\omega^2+a)}+\frac{\omega^2(a+b\omega+c\omega^2)}{(c\omega^2+a+a\omega)}=\omega+\omega^2=-1$

**96.** (d) The $n^{\text{th}}$ roots of unity are given by

$z_k=e^{\frac{i2\pi(k-1)}{n}}, \quad (k=1,2....,n)$

$\therefore |z_k|=\left|e^{\frac{i2\pi(k-1)}{n}}\right|=1$ for all $k=1,2,.....,n$

$\Rightarrow |z_k|=|z_{k+1}|$ for all $k=1,2.....,n$

**97.** (a) $2^{15}\left[\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{15}}{(1-i)^{20}}+\frac{\left(\frac{-1}{2}-\frac{i\sqrt{3}}{2}\right)^{15}}{(1+i)^{20}}\right]$

$= 2^{15}\left[\frac{\omega^{15}}{(1-i)^{20}}+\frac{\omega^{30}}{(1+i)^{20}}\right]=2^{15}\left[\frac{1}{(1-i)^{20}}+\frac{1}{(1+i)^{20}}\right]$

**98.** (d) As $\frac{-1+i\sqrt{3}}{2}=\omega$ and $\frac{-1-i\sqrt{3}}{2}=\omega^2$

$\therefore (\omega)^{20}+(\omega^2)^{20}=\omega^{18}.\omega^2+\omega^{39}.\omega=\omega^2+\omega=-1$

**99.** (d) $z+z^{-1}=1 \Rightarrow z^2-z+1=0 \Rightarrow z=-\omega$ or $-\omega^2$

For $z=-\omega,\ z^{100}+z^{-100}=(-\omega)^{100}+(-\omega)^{-100}$

$= \omega+\frac{1}{\omega}=\omega+\omega^2=-1$

For $z=-\omega^2$, $z^{100}+z^{-100}=(-\omega^2)^{100}+(-\omega^2)^{-100}$

$=\omega^{200}+\frac{1}{\omega^{200}}=\omega^2+\frac{1}{\omega^2}=\omega^2+\omega=-1.$

**100.** (c) $x^4-x^3+x-1=0 \Rightarrow x^3(x-1)+1(x-1)=0$

$x-1=0$ Or $x^3+1=0 \Rightarrow x=1,-1,\frac{1+\sqrt{3}i}{2},\frac{1-\sqrt{3}i}{2}$

so its real roots are 1 and – 1.

**101.** (d) $225+(3\omega+8\omega^2)^2+(3\omega^2+8\omega)^2$

$= 225+(5\omega^2-3)^2+(5\omega-3)^2$

$= 225+18-5(\omega+\omega^2)$

$= 225+18-5(-1)=225+18+5=248.$

**102.** (b) $\sinh ix=i\sin x$ .

**103.** (a) $\Delta=(\omega^{3n}-1)+\omega^n(\omega^{2n}-\omega^{2n})+\omega^{2n}(\omega^n-\omega^{4n})$

$\Delta=(1-1)+0+\omega^{2n}[\omega^n-(\omega^3)^n\omega^n]$

$\Delta=0+0+0=0$ .